

Investigations of the spall rupture of lead under intense explosive and shock loads were performed in [1-4]. Available is a substantial (by more than an order of magnitude) excess of the dynamic strength of the material being realized during spall as compared with the strength under quasistatic uniaxial tension and torsion conditions [5, 6]. This is in good agreement with the tendency observed in [1] to an increase in the ratio between the strength realized during spall and the static strength as the material plasticity rises.

Preliminary results of investigations on the spall rupture of lead in the $-196-300^{\circ}\text{C}$ temperature range were presented in [3], where the critical loading levels corresponding to macroscopic rupture of the material were determined. In this paper, additional experiments are performed in order to determine the loading levels corresponding to generations of spall microdamage in the material, as are a detailed metallographic analysis of the specimens in order to determine the nature of the rupture, as well as computational estimates of the viscosity of lead under spall rupture conditions. The scheme for performing the experiments was presented in [7], and the metallographic analysis method was used to determine the nature of the spall rupture of steel in a broad temperature range [8].

Specimens of lead C2 of 4-mm thickness and 40-mm diameter were fastened to a 12-mm-thick copper shield. Mechanical pulse loading of the specimens was by impact of a 4-mm-thick aluminum plate on the copper shield. The computationally estimated value of the pressure in the loading compression pulse, whose reflection from the specimen free surface in the form of a tensile pulse indeed resulted in spall rupture of the lead, was set in correspondence to the known impact velocity. To produce reflection conditions of a tensile pulse with a negative pressure less than 0.6 GPa in the lead, a polytetrafluoroethylene plate was pressed to the lead specimen. The known material shock adiabat in the form of linear $D-u$ relationships used for the computational estimates are presented in [4, 7], for instance. The characteristic loading time was estimated approximately at 1.3-1.5 μsec . The legitimacy of the approximate computation used for the loading conditions is confirmed experimentally by using a manganin pressure sensor [9]. The results of recording the compression loading pulse by the manganin pressure sensor showed that the computational estimates do not emerge beyond the limits of error in the sensor, and the characteristic pulse duration at half the height is about 1.5 μsec .

Results of the experiments conducted are given in Fig. 1, where the degree of spall damage of the material, determined by observing a longitudinal section of the specimen under a microscope, is set in correspondence with specific loading conditions. In conformity with the rupture pattern observed, the degree of spall damage of the material is provisionally subdivided into five gradations: 1) total spall rupture, exfoliation of the whole spall layer; 2) partial spall rupture, exfoliation of part of the spall layer; 3) intensive micro-rupture of the material, the presence of a significant number of damages in the form of isolated or coalescing pores in the spall zone; 4) weak material microrupture, the presence of a moderate number of isolated pores in the spall zone or in its individual parts; 5) actual absence of spall rupture. It should be noted that single and comparatively shallow damage was observed in the specimen cut section even in case 5. The loading levels corresponding to disturbance of the macroscopic and microscopic wholeness of the material are marked in Fig. 1 by solid and dashed lines, respectively. The exterior view of the specimens tested at temperatures of about 0°C is presented in Fig. 2. The value of the pressure in the loading pulse is 0.76, 0.78 and 0.81 GPa, respectively.

For the metallographic investigations, the longitudinal section of the specimen was made by a sharp knife by using a manual press. In this case the material was practically undeformed, and a significant number of microdamages in the spall zone could be observed at once

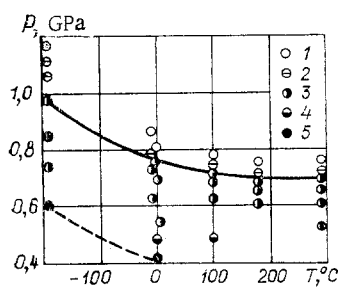


Fig. 1



Fig. 2

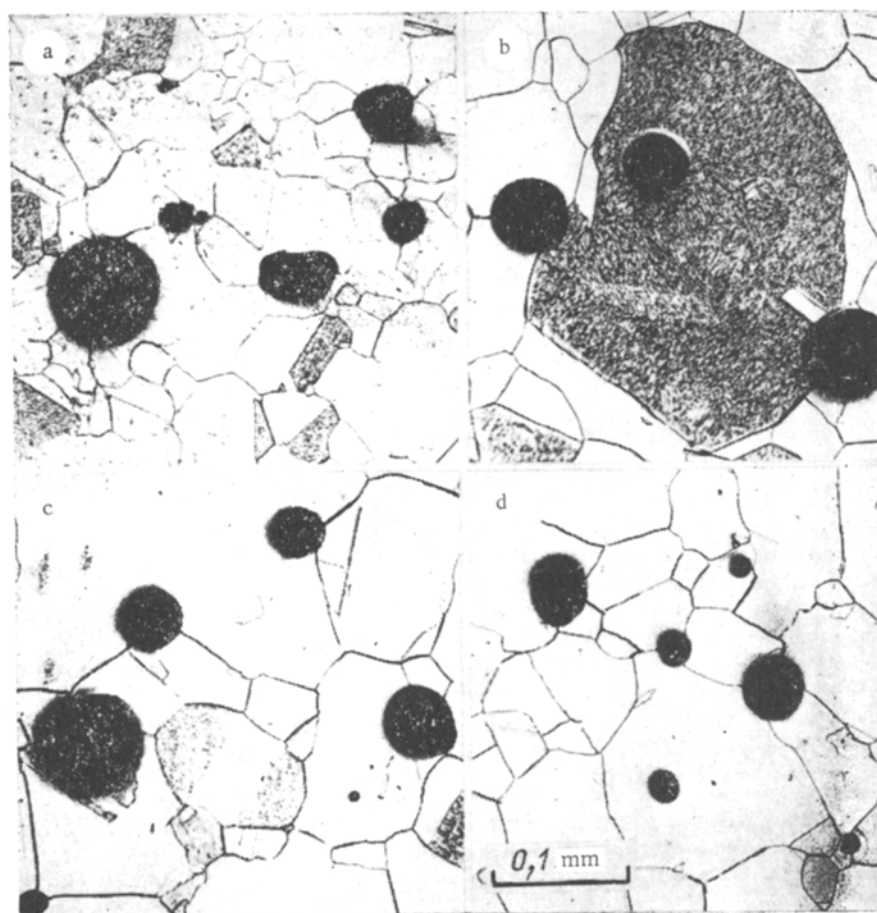


Fig. 3

on the surface of the section of the seemingly undamaged specimen. Subsequent chemical polishing of the section was performed in a reagent containing equal quantities of acetic acid and hydrogen peroxide. The mean grain size of the specimens under investigation was 80-90 μm , where no changes in the grain size were noted upon heating to 300°C. Fragments from the spall damage zone of lead are presented in Fig. 3 for different test temperatures. The notation a-d corresponds to the temperature -196, 0, 100 and 300°C and the pressures 0.75 GPa for $T = -196^\circ\text{C}$ and 0.6 for the rest. It is seen that the initial stage of spall rupture of lead is related to the generation and growth of pores of almost spherical shape in the whole temperature range under investigation. An analogous spall rupture was observed in [10] for aluminum and copper at a normal test temperature. As a rule, pore generation occurs at

sites of possible microstress concentration such as grain boundaries and twins, but is also observed within the grains at sites apparently not discernible by the optical microscopy of structure defects. Specimens loaded by the 0.6 GPa pressure pulse at 0-300°C temperatures were inspected in more detail at a magnification of $\times 1000$. The minimal and maximal pore size observable at all the test temperatures were approximately 1 and 100 μm . At higher loading levels, a certain increase in the maximum size of individual pores is observed, but in connection with the increase in their density the main process responsible for rupture is coalescence. The difference for the temperature -196°C is related just to a certain rise in the appropriate loading levels. The majority of spall damage is concentrated in a zone about 1 mm wide for all the test temperatures.

Under quasistatic uniaxial tension conditions with the least possible loading time $t = 10^{-2}$ sec, values of the strength of lead that equal 18 and 30 MPa, respectively, are obtained in [5, 6]. The observable excess over the strength of lead realized during spall as compared with the static test results is due both to the significant reduction loading time and the hindrance to plastic deformation of the material under triaxial tension conditions. Hindrance to plastic deformation makes pore formation in a plastic material difficult, and results in a significant rise in the strength under static loading conditions [11]. Under dynamic shockwave loading conditions of such a plastic material as lead, a state close to hydrostatic tension is realized in the negative pressure zone, i.e., possessing the greatest degree of hindrance to macroscopic plastic deformation. Therefore, plastic deformation of the material at a microlevel specifying the formation of dislocation clusters at such obstacles as the grain boundaries and twins has the governing role in pore generation.

The loading levels corresponding to macroscopic rupture of lead during spall were estimated in [1, 2, 4] from the thickness of the spall layer and the shockwave profile emerging on the specimen free surface as determined experimentally [1] or computationally [2, 4]. Values obtained in this manner for the rupturing loads have a tendency to increase as the characteristic loading time decreases. It must be noted that the value of the load corresponding to the macroscopic rupture of the material is only the upper bound in connection with not taking account of the negative pressure relaxation in the tension pulse due to the generation and growth of a significant number of microdamages which have been formed at the lower loading levels.

The nature of the spall rupture of lead observable in the research is due to generation and growth of a large number of specific pores. The range of sizes of observable pores under specific test conditions is $\sim 1-100 \mu\text{m}$. It is expedient to carry out an approximate computation of the growth of isolated pores under loading conditions corresponding to a moderate level of spall damage of the material, which can yield additional information about the resistance of material to deformation at the high strain rates being realized during spall.

As was noted in [12], the material strain rate in a pore layer exceeds 10^6 sec^{-1} in viscous dynamic rupture. At such a strain rate, the viscous component of the resistance to strain becomes governing even for materials with relatively low viscosity. For lead which possesses a low value of the plastic component of the resistance to strain, an approximate computation of the isolated pore growth can be executed completely correctly by using the model of a viscous incompressible fluid.

We use the approximate analytic method of computing bubble growth in a viscous fluid caused by a short-range negative pressure pulse, which was proposed in [13]. Let us briefly note the crux of the method to be used. The governing equation for the bubble radius $R(t)$ has the form

$$R \frac{d^2 R}{dt^2} + \frac{3}{2} \left(\frac{dR}{dt} \right)^2 + \frac{4\mu}{\rho R} \frac{dR}{dt} = \frac{1}{\rho} P(t), \quad (1)$$

where ρ and μ are the fluid density and coefficient of viscosity. The initial conditions and loading pressure pulse are taken in the form

$$R(0) = R_0, \quad \frac{dR}{dt}(0) = 0, \quad P(t) = \begin{cases} P_0, & 0 < t < t_0, \\ 0, & t > t_0. \end{cases}$$

Introduction of the dimensionless coordinates and parameters

$$r = R/R_0, \quad \tau = t/t_0, \quad \eta = 4\mu t_0^2 / \rho R_0^2, \quad p = P_0 t_0^2 / \rho R_0^2$$

reduces (1) to the form

$$r \frac{d^2 r}{d\tau^2} + \frac{3}{2} \left(\frac{dr}{d\tau} \right)^2 + \frac{\eta}{r} \frac{dr}{d\tau} = p [H(\tau) - H(\tau - 1)], \quad (2)$$

$$r(0) = 1, \quad \frac{dr}{d\tau}(0) = 0,$$

where $H(\tau)$ is the Heaviside step function. For $0 < \tau < 1$ the first integral in (2) is given by the approximate formula

$$dr/d\tau = -\eta/3r + [(\eta/3r)^2 + (2/3)p(1 - 1/r^3)]^{1/2}, \quad (3)$$

while for $\tau > 1$ the first integral will be

$$dr/d\tau = b/r^{3/2} - 2\eta/r, \quad (4)$$

where b is the constant of integration determined for $4 = r_1$, and r_1 is the bubble radius at $\tau = 1$. From the condition $dr/d\tau = 0$ the maximal radius of the expanding bubble r_2 is determined

$$r_2 = b^2/4\eta^2. \quad (5)$$

In the case $p/\eta^2 \ll 1$ for not very large r (3) can be written approximately in the form

$$dr/d\tau \approx (p/\eta)r(1 - 1/r^3),$$

which, eliminating the case of small r also, yields the approximate value of r_1 :

$$r_1 \approx \exp(p/\eta). \quad (6)$$

It is seen from (6) that the necessary condition for the origination of cavitation in a viscous fluid is the condition $p/\eta > 1$, from which the upper bound for the viscosity of lead can be obtained at once under spall rupture conditions $p/\eta = P_0 t_0 / 4\mu > 1$. For $P_0 = 0.6$ GPa and $t_0 = 10^{-6}$ sec the upper bound of μ yields $1.5 \cdot 10^2$ Pa·sec.

If the value of p/η is large under the assumption of an increase in r to a value much greater than unity, (3) can be written approximately in the form

$$\frac{1}{2} \frac{dr^2}{d\tau} = -\frac{\eta}{3} + \left[\left(\frac{\eta}{3} \right)^2 + \frac{2}{3} pr^2 \right]^{1/2}. \quad (7)$$

Integrating (7) we find

$$\frac{2p}{\eta} = \left(1 + \frac{6p}{\eta^2} r_1^2 \right)^{1/2} - \left(1 + \frac{6p}{\eta^2} \right)^{1/2} + \ln \frac{\left(1 + \frac{6p}{\eta^2} r_1^2 \right)^{1/2} - 1}{\left(1 + \frac{6p}{\eta^2} \right)^{1/2} - 1}. \quad (8)$$

Taking account of the condition $p/\eta^2 \ll 1$, we simplify (8) somewhat

$$\left(1 + \frac{6p}{\eta^2} r_1^2 \right)^{1/2} + \ln \left[\left(1 + \frac{6p}{\eta^2} r_1^2 \right)^{1/2} - 1 \right] - \frac{2p}{\eta} - 1 + \ln \frac{\eta^2}{3p} = 0. \quad (9)$$

The further simplifications presented in [13] become incorrect for conditions of the problem considered in this paper. Nevertheless, the determination of r_1 for a specific value of $p = 2 \cdot 10^5$ and a number of values of p/η , for instance in the 2-20 range, reduces to an elementary numerical solution of the transcendental equation (9). Having determined r_1 in such manner, we find $v = dr/d\tau$ at this radius from (3), then we determined b from (4), and finally we obtain the value of r_2 from (5). The dependences obtained for r_1 , r_2 , and v (curves 1-3) are presented in Fig. 4.

The strong dependence of r_2 on the parameter p/η observed in Fig. 4 permits a sufficiently correct estimation of the magnitude of the dynamic viscosity coefficient for lead under spall rupture conditions. The results of a metallographic analysis performed on the specimens show that $r_2 \approx 100$. It must be noted that the plastic component of the resistance to strain induces a governing contribution to the retardation of the pore being expanded in the final stage of inertial pore expansion ($\tau > 1$). This should result in a reduction in the real value of r_2 , corresponding to the specific value of p/η . Therefore, the value $p/\eta = 5$ of the parameter apparently yields a completely satisfactory estimate of the viscosity of lead $\mu \approx 30$ Pa·sec, and the range $p/\eta = 4-6$ already contains the real value of μ with a high degree of confidence since the value of r_2 for $p/\eta = 6$ is 250. Estimation of the magnitude of the radial strain rate component yields

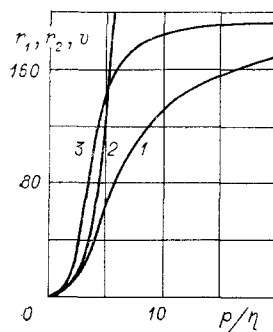


Fig. 4

$$\dot{\epsilon}_r \approx \frac{d\left(\frac{dR}{dt}\right)}{dR} = \frac{1}{t_0} \frac{d\left(\frac{dr}{d\tau}\right)}{dr} \approx \frac{1}{t_0} \frac{p}{\eta} = 5 \cdot 10^6 \text{ sec}^{-1}.$$

Determination of the dynamic viscosity coefficient of lead under shockwave compression was carried out in [14, 15]. A method based on observing the development of small perturbations on a shock front with pressure 35 GPa was used in the first, while the viscosity was determined in the second at pressures an order less by means of an experimentally determined dependence of the displacement of a fixed line during oblique collision of plates. The approximate values of μ obtained are $4 \cdot 10^3$ and $5 \cdot 10^3$ Pa·sec, i.e., actually agree, and the strain rate was estimated at 10^7 sec^{-1} in [14].

In connection with the complexity of the phenomena underlying the regularities of the viscous behavior of solids, we note just certain possible factors that can exert influence on these regularities. Thus, it is noted in [14] that at a pressure of ~ 30 GPa the viscosity of aluminum is independent of the strain rate in the $\sim 10^5$ - 10^7 sec^{-1} range. Comparing the results in [14] and [15] does not indicate also that the pressure exerts a governing influence on the viscosity of metals in this strain-rate range, while the interrelation between viscosity and pressure in water is observed clearly in [16]. Under the conditions of viscous spall rupture of aluminum and copper [10], the pore sizes observed were described by a formula similar to (6), where the value of μ used for both metals was 20 Pa·sec. The results of this paper indicate that temperature variation within broad limits does not exert substantial influence on the value of μ determined for lead. A possible influence on the value determined for the viscosity of lead as, however, of other plastic metals also, is apparently exerted by the scale of the phenomenon under consideration. In considering the expansion of an isolated pore, the viscous flow is constrained by one or several nearby grains, i.e., the value obtained for the viscosity coefficient in pure form determines the viscosity of the real crystal structure due to dynamic deceleration of the moving dislocations. In the case of a macroscopic examination of the phenomenon a higher value of the dynamic viscosity can be associated partially with the associated flow of a large number of crystallites being deformed during the flow.

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OSCILLATIONS GENERATED IN THE COMPRESSION OF A
VISCOELASTIC BODY

G. I. Burdé and T. M. Burdé

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Studies of the various kinds of instability that evolve in processes of deformation of viscoelastic materials have been reported in a large number of papers (see [1] and [2-7]). Mainly elongational flows of incompressible liquids have been investigated in these studies. As a rule, the inertial terms in the equation of motion are neglected in the analysis of stability.

It is shown below that in deformations of compressible viscoelastic bodies another type of instability is possible, namely oscillations induced by the bulk elasticity of the material and driven up in hydrostatic compression of the sample as a result of inertial interaction of the disturbances with the main flow. The conditions for growth of the oscillations are determined from the linearized small-perturbation equations; on the basis of a nonlinear analysis, the nature of the excitation is investigated, and the amplitude of the oscillations is determined.

Calculations are carried out for the case of uniform planar deformation and ideally smooth and rigid bounding surfaces. These assumptions clearly do not play a vital role in the investigated instability mechanism.

1. Let us consider a rectangular compressible viscoelastic body (Fig. 1) bounded by smooth rigid planes. The planes $x_1 = \pm L(t)$ move symmetrically relative to one another with a velocity $U = dL/dt$, and the planes $x_2 = \pm R(t)$ do likewise with a velocity $V = dR/dt$. The velocities U and V can be either positive (extension) or negative (compression) and are assumed to be constant, so that the dimensions of the sample vary with time according to the linear law

$$L = L_0 + Ut, \quad R = R_0 + Vt. \quad (1.1)$$

The equations describing the isothermal flow of the sample material have the form (repeated indices signify summation)

$$\partial \rho / \partial t + \partial(\rho v_k) / \partial x_k = 0; \quad (1.2)$$

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